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## Analogy between momentum and heat transfer in liquid-solid fluidized beds

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### Abstract

Wall-to-bed heat transfer in particulate fluidized beds of spherical particles was studied. Experiments were performed using spherical glass particles of 0.80 - 2.98 mm in diameter with water in a 25.4 mm I.D. copper tube equipped with a steam jacket.

Heat transfer data related to the fluid-particle interphase drag coefficient were obtained and compared with previous results for wall-to-bed mass transfer in fluidized beds [Bošković et. al., Powder Technol., 79 (1994) 217].

All the data for momentum, heat and mass transfer in particulate fluidized beds of spherical particles, showed the existence of an analogy among these three phenomena.

**Keywords:** fluidized bed, momentum transfer, heat transfer, mass transfer, analogy

Heat transfer in liquid-solid systems is a very important parameter to be considered in the design of equipment for different industrial applications. The design of equipment with fluidized beds is mainly based on knowledge of the hydrodynamics and heat transfer between

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wall and fluidized beds. Numerous studies on wall-to-liquid heat transfer in particulate fluidized beds have been realized [1 - 6]. In these studies, the influence of different parameters, such as liquid velocity, particles size and voidage on heat transfer in fluidized beds were investigated.

The subject of the present research was the effect of particles on the wall-to-bed heat transfer. An attempt was made to establish analogy between heat transfer coefficients and fluid-particle interphase drag coefficient.

In a previous study, Bošković et al. [7] found that an analogy between mass and momentum transfer in liquid-solid fluidized beds exists. In addition, the dimensionless mass transfer factor in liquid-fluidized beds of active and inert particles and the dimensionless drag coefficient [8] were shown to be the same.

$$j_D^* = \beta^* \quad (1)$$

where  $j_D^*$  and  $\beta^*$  are given by,

$$j_D^* = \frac{j_D - j_{D_1}}{j_{D_{mf}} - j_{D_1}} \quad (2)$$

$$\beta^* = \frac{\beta}{\beta_{mf}} = 1 - C_2 + \frac{1}{\lambda} \sqrt{1 - \left( \lambda \frac{\varepsilon - \varepsilon_{mf}}{1 - \varepsilon_{mf}} + C_1 \right)^2} \quad (3)$$

Wall-to-bed heat transfer in particulate fluidized beds of spherical particles was studied. Experiments were performed using spherical glass particles of 0.80, 1.11, 1.94 and 2.98 mm in diameter that were fluidized with water in a 25.4 mm I.D. copper tube equipped with a steam jacket. The schematic diagram of the experimental systems is shown in Fig. 1.

The fluid bed (a, Fig. 1) was the 27.4/25.4 mm OD/ID, 1360 mm long copper tube, equipped with a 700 mm long steam jacket (b). The heating section (b) was located far enough (320 mm) from the inlet nozzle (d) of the fluid bed. Water was introduced at the bottom of the bed.

The pressure gradient was measured using piezometers (i) and temperature was measured using Ni–Cr thermocouples. The wall temperature was determined at two points, at the inlet and at the exit of the heating zone;  $T_{01}$ ,  $T_{02}$ . The junction points were filled with tin at about 0.2 mm from the inside of the tube wall, as shown schematically in Fig. 1 (detail A). The temperature of the fluidized bed was measured with thermocouples located along the tube axis ( $T_{\infty 1}$ ,  $T_{\infty 2}$ ). It was assumed that at the inlet and at the outlet of the heating zone, the particles and the fluid had the same temperature [2]. The heat transfer coefficient in the fluidized bed was calculated as [3]:

$$\alpha = \frac{G_f c_{pf} (T_{\infty 2} - T_{\infty 1})}{D_c \pi L_H \Delta T_{ln}} \quad (4)$$

The mean logarithmic temperature difference in Eq. (4) is defined as:

$$\Delta T_{ln} = \frac{(T_{0,2} - T_{\infty,2}) - (T_{0,1} - T_{\infty,1})}{\ln \frac{(T_{0,2} - T_{\infty,2})}{(T_{0,1} - T_{\infty,1})}} \quad (5)$$

Fig. 1

A total of 156 data points for heat transfer coefficients were collected in the experimental runs. The characteristics of the particles and the employed range of experimental conditions are given in Table 1. Water was used as the fluidizing medium, and its characteristics were determined as temperature dependencies [9]:  $\mu = f(T_m)$ ,  $\rho_f = f(T_m)$  and  $c_{pf} = f(T_m)$ .

Table 1.

The variation in the wall-to-bed heat transfer factor and the dimensionless drag coefficient in dependence on the bed voidage for the 2.98 mm diameter particles are

illustrated in Fig. 2 from which it is evident that the heat transfer factor decreased with increasing bed voidage. It could be registered that this plot is very similar to the variation of the dimensionless fluid-particle interphase drag coefficient with bed voidage.

Fig. 2

The present experimental data for heat transfer factor, analogous to the work of Bošković et al. [7], can be represented as (Eq. 1):

$$j_H^* = \beta^* \quad (6)$$

where  $j_H^*$  is

$$j_H^* = \frac{j_H - j_{H_1}}{j_{H_{mf}} - j_{H_1}} \quad (7)$$

If Eq. (3) is substituted into Eq. (7) then:

$$\frac{j_H - j_{H_1}}{j_{H_{mf}} - j_{H_1}} = 1 - C_2 + \frac{1}{\lambda} \left[ \sqrt{1 - \left( \lambda \frac{\varepsilon - \varepsilon_{mf}}{1 - \varepsilon_{mf}} + C_1 \right)^2} \right]^{1/2} \quad (8)$$

The data for the heat transfer factor at the terminal velocity,  $j_{H_1}$ , and for the heat transfer factor at minimum fluidization velocity,  $j_{H_{mf}}$ , could be acquired by extrapolating experimental data that are given as the dependency of the heat transfer factor,  $j_H$ , on the dimensionless fluid-particle interphase drag coefficient,  $\beta^*$ . The parameters  $j_{H_1}$  and  $j_{H_{mf}}$  are determined for both:  $\beta/\beta_{mf} = 0$  (terminal velocity) and for  $\beta/\beta_{mf} = 1$  (minimum fluidization), as can be seen in Fig. 3.

Fig. 3

All the data for the dimensionless  $j_H^*$  factor acquired in the present experimental runs, the experimental data for the dimensionless  $j_D^*$  factor from the previous work of Bošković et al. [7], and the data for the dimensionless fluid-particle interphase drag coefficient  $\beta^*$  are plotted against the dimensionless bed voidage  $\varepsilon^* = (\varepsilon - \varepsilon_{mf}) / (1 - \varepsilon_{mf})$  in Fig. 4.

As can be seen from Fig. 4, the values of  $j_H^*$ ,  $j_D^*$  and  $\beta^*$  are practically the same in the range of the investigated conditions, clearly indicating an analogy among these phenomena. From all the presented results, it could be concluded that all data for fluidized bed could be described by the following correlation:

$$j_H^* = j_D^* = \beta^* \quad (9)$$

The mean absolute deviation between the experimental data of the dimensionless heat transfer factor and Eq. (3) is 18.33%.

Fig. 4

The analogy established in this work enables the application of the proposed model for the determination of the fluid-particle interphase drag coefficient (Eq. 3) and of heat and mass transfer factors in liquid-solid fluidized beds (Eq. 9).

## Nomenclature

$C_1$	variational constant in Eq. (3)
$C_2$	variational constant in Eq. (3)
$c_{pf}$	specific heat of fluid, J/(kgK)
$d_p$	particle diameter, m

$D_c$	column diameter, m
$G_f$	fluid mass flowrate in the column, kg/s
$j_H$	$Nu/Re Pr^{1/3}$ , heat transfer factor
$j_{H_t}$	heat transfer factor at the terminal velocity
$j_{H_{mf}}$	heat transfer factor at minimum fluidization
$j_H^*$	$(j_H - j_{H_t}) / (j_{H_{mf}} - j_{H_t})$ , dimensionless $j_H$ factor
$j_D$	$Sh/Re Sc^{1/3}$ , mass transfer factor
$j_{D_t}$	mass transfer factor at the terminal velocity
$j_{D_{mf}}$	mass transfer factor at minimum fluidization
$j_D^*$	$(j_D - j_{D_t}) / (j_{D_{mf}} - j_{D_t})$ , dimensionless $j_D$ factor
$L_H$	length of heating zone, m
$T$	temperature (K)
$T_0$	temperature of the column wall (K)
$T_\infty$	fluid temperature (K)
$T_m$	$(T_{\infty,1} + T_{\infty,2}) / 2$ , means temperature (K)
$U_t$	terminal velocity, m/s
$U_{mf}$	superficial fluid velocity at minimum fluidization, m/s
$V$	water flowrate at the column inlet ( $m^3/s$ ) (Fig. 1)

*Greek letters*

$\alpha$	heat transfer coefficient, $W/(m^2K)$
$\beta$	fluid-particle interphase drag coefficient, $kg/m^4$
$\beta_{mf}$	fluid-particle interphase drag coefficient at minimum fluidization, $kg/m^4$

$\beta^*$   $\beta/\beta_{mf}$ , dimensionless fluid-particle interphase drag coefficient

$\varepsilon$  averaged voidage in the fluidized bed

$\varepsilon_{mf}$  voidage at minimum fluidization

$\varepsilon^*$   $(\varepsilon - \varepsilon_{mf})/(1 - \varepsilon_{mf})$ , dimensionless bed voidage

$\lambda$  variational constant in Eq. (3)

$\mu$  fluid viscosity, Pa·s

$\rho_f$  fluid density, kg/m<sup>3</sup>

$\rho_p$  particle density, kg/m<sup>3</sup>

$\delta_{sr.}$  the mean absolute deviation  $(=100 \cdot \frac{1}{n} \sum_{i=1}^n \frac{|\Gamma_{exp.} - \Gamma_{calc.}|}{\Gamma_{calc.}})$ , %

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Fig. 1. Schematic diagram of the experimental fluidization system: (a) - fluid bed copper column, 25.4 mm i.d.; (b) - heating section, 700 mm in length; (c) - steam generator, 30 kW; (d) - inlet nozzle, 20 mm i.d.; (e) - screen; (f) - distributor; (g) - fluidized bed; (h) - overflow; (i) - pressure taps; (j) - flow meter; (k) - valve; (m) - Ni–Cr thermocouple; (n) - copper tube 8/6 mm; (o) - jacket wall; (p) - column wall; (q) - thermal insulation; V - inlet flow rate.

Fig. 2. The experimental data for the heat transfer factor and dimensionless drag coefficient vs. voidage, for a fluidized bed ( $d_p = 2.98\text{mm}$ ).

Fig. 3. Determination of  $j_H$  at the minimum fluidization and terminal using the relationship  $j_H$  vs.  $\beta^*$ .

Fig. 4. A comparison of the data for heat transfer against mass transfer [7] and the friction factor in a fluidized bed.

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Table 1. Particle characteristics and range of experimental conditions (at 293.15K).

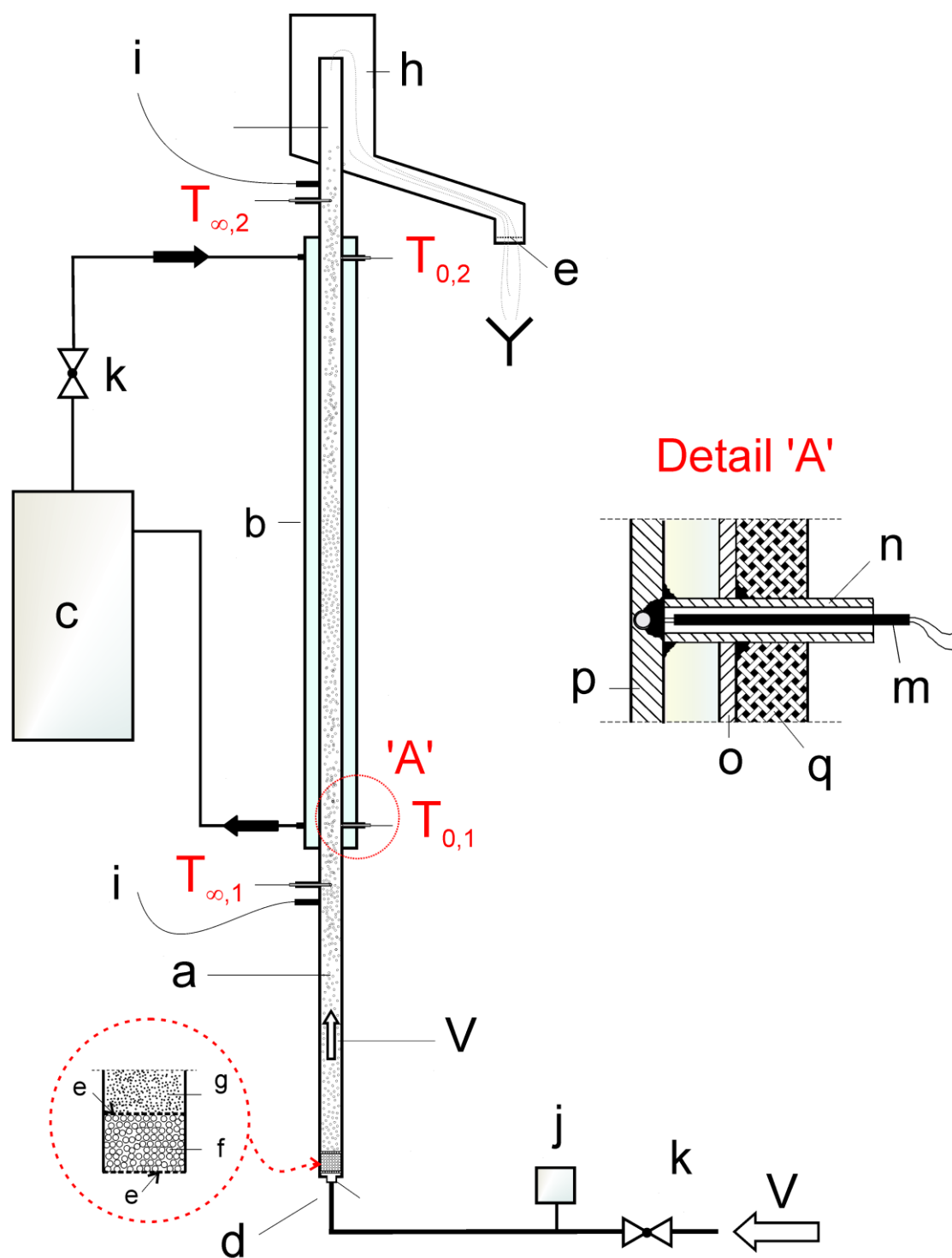


Fig. 1

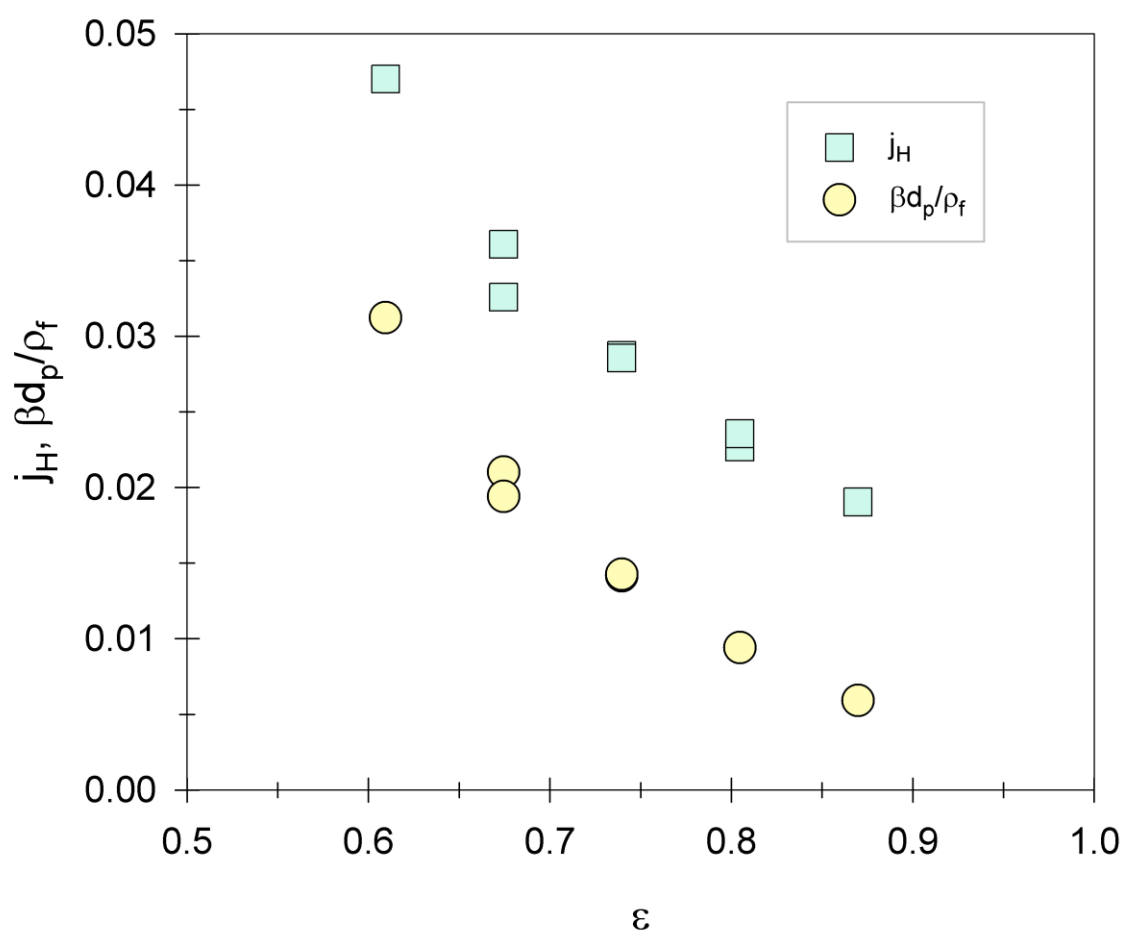


Fig. 2

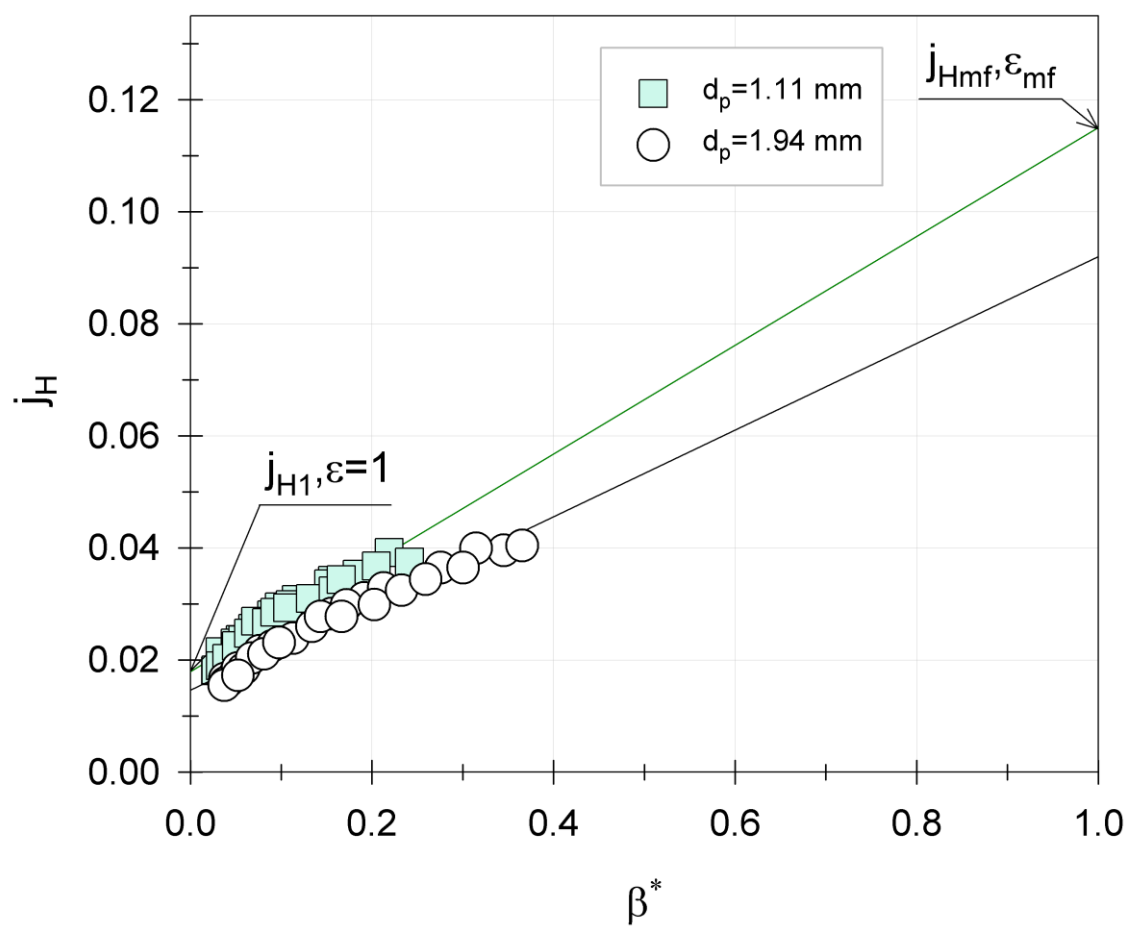


Fig. 3

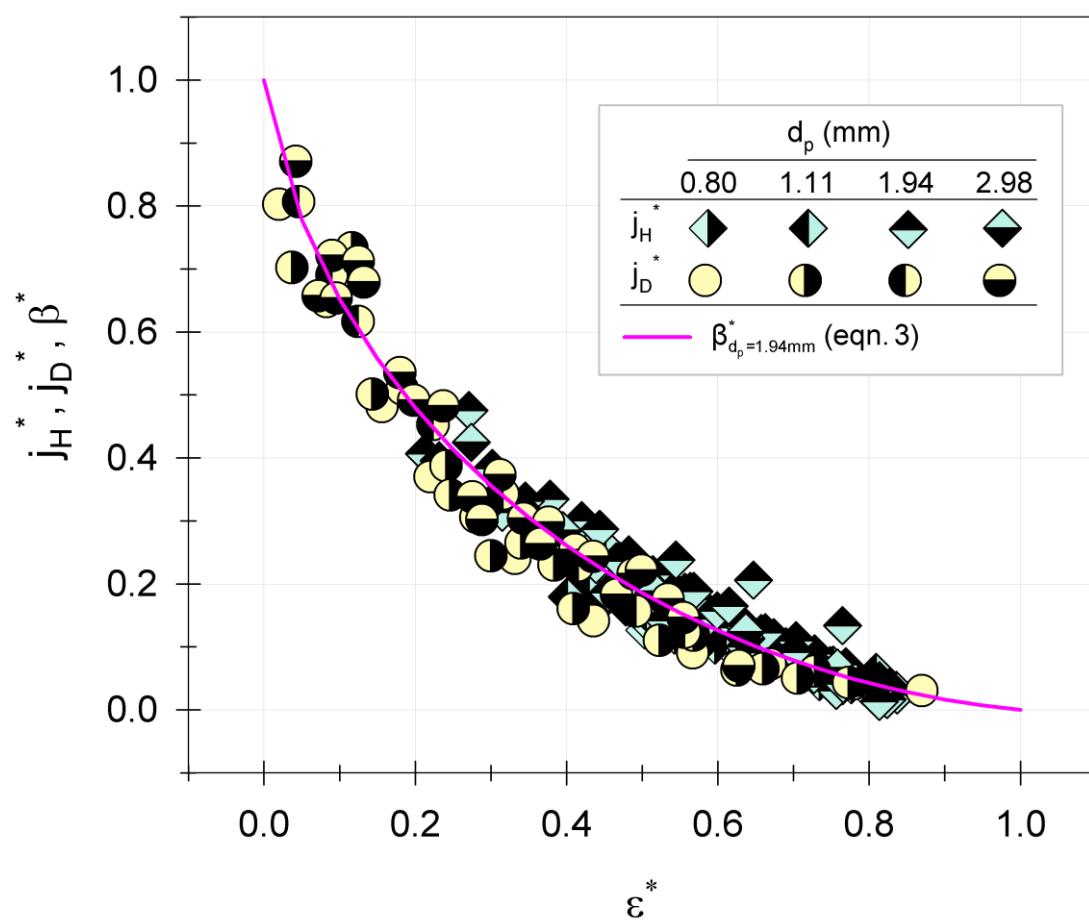


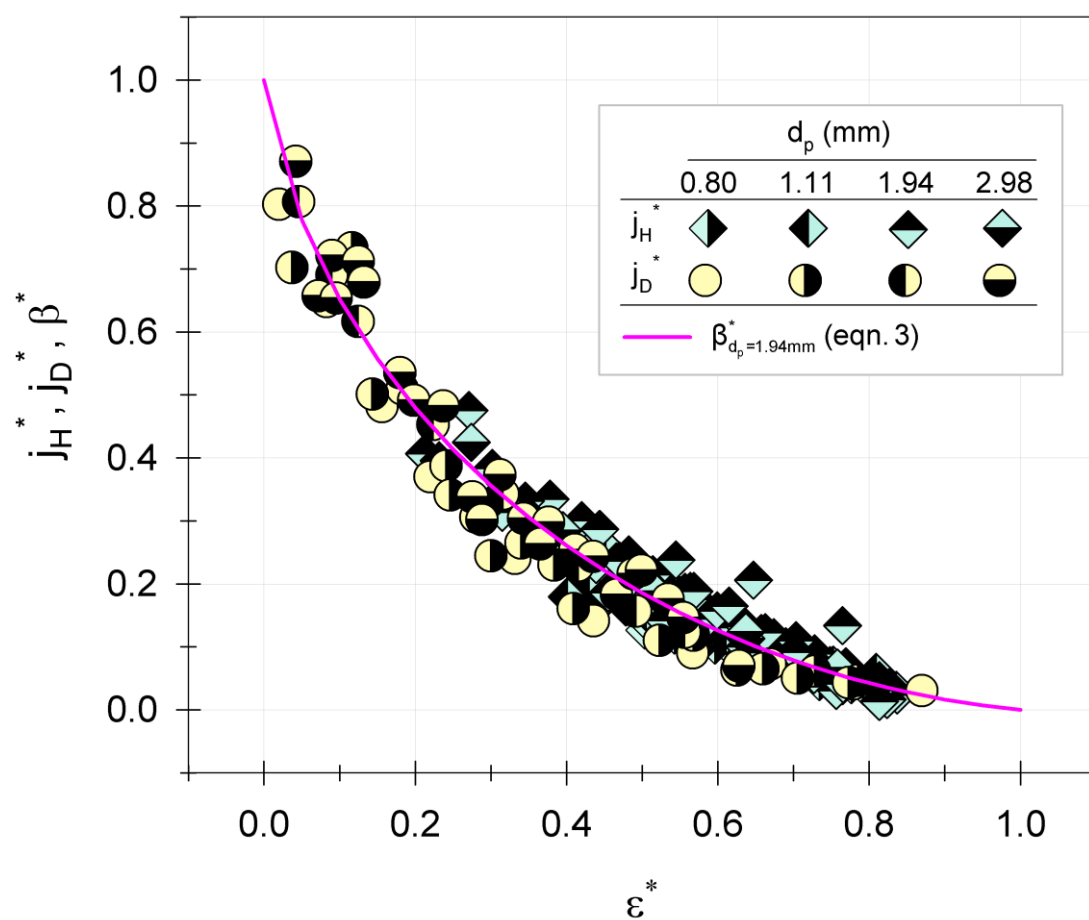
Fig. 4

Table 1.

$d_p$ (mm)	0.80	1.11	1.94	2.98
$\rho_p$ (kg/m <sup>3</sup> )	2923	2641	2507	2509
$U_t$ (m/s) <sup>*</sup>	0.148	0.185	0.299	0.370
$U_{mf}$ , m/s <sup>*</sup>	0.008	0.013	0.028	0.043
$U/U_{mf}$	5.224-17.401	3.797-9.417	1.679-6.311	1.883-4.080
$\varepsilon_{mf}$	0.398	0.416	0.447	0.462
$\varepsilon/\varepsilon_{mf}$	1.638-2.393	1.569-2.175	1.126-2.039	1.319-1.883

<sup>\*</sup> Calculated from Kunii and Levenspiel [10]





Graphical abstract

**Highlights:**

- Fluid-particle interphase drag coefficient in liquid-solid fluidized beds.
- Wall-to-bed heat and mass transfer in particulate fluidized beds of spherical particles.
- Analogy between momentum, heat and mass transfer in particulate fluidized beds.